

Research on Combination of Rough Set and Seeking for Multi-objective Decision-Making

Zhenghua Cui, Jian Huang, Jianguo Hao, Mingguang Gao, and Jiangtao Kong

Abstract—Considering the technical indicator identification problem in preliminary weaponry research, the paper provides a novel approach by combining operational effectiveness with multi-objective optimization algorithms (MOO). Through using effectiveness evaluation approaches based on Rough Set Theory and having the set of 20 kinds of gunships, which are widely equipped nowadays, as the knowledge base, the paper first evaluates several performance factors and reaches the decision schema satisfying completion requirement after two times of filtering. Then the result is optimized using MOO algorithms under different weights over objectives. During the process, two phrases of filtering are applied, thus reducing the problem complexity. Meanwhile, the function of evaluating is also modified to improve the intuition and accuracy of results. This paper realizes the combinatory application of fuzzy set theory and MOO in the domain of operational effectiveness evaluation.

Index Terms—Rough sets, fuzzy multi-objective, configuration schemes, evaluate, decision-making.

I. INTRODUCTION

It has numerous of diverse factors with the problem of researching and developing weapons along with the other equipment in advance. On the one hand, the results of operational effectiveness evaluation should meet the requirements of a specific task, on the other hand, the configuration scheme should take restrictions into consideration, such as the situation of the present technology, financial and time aspects. The measure of evaluating the operational effectiveness which is based on rough set theory has advantages to deal with the fuzzy data. In addition, seeking for fuzzy multi-objective decision-making could play an important role in the process of achieve the optimal configuration scheme.

II. METHODS OF EVALUATING AND SEEKING FOR DECISION-MAKING

A. Evaluating Operational Effectiveness Based on Rough Sets Theory

Rough Set theory is proposed by the Polish mathematician Z. Pawlak in 1982, is a new tool to deal with vagueness and uncertainty data [1]. The rough set theory has been found to have quite successful applications in the field of evaluating effectiveness. The primary coverage includes the

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Zhenghua Cui, Jian Huang, Jianguo Hao, and Jiangtao Kong are with the National University of Defense Technology, Changsha, Hunan, China (e-mail: cocolovelife@outlook.com, Huang_jian@139.com, hla2000@tom.com, Jiangtao_Kong@163.com).

Mingguang Gao is with the Beijing Institute of Technology, Beijing, China (e-mail: sunnyangelgmg@outlook.com).

interpretation of the connection between the inaccurate data as well as discovering the relationship between objects and attributes [2].

B. Seeking For Fuzzy Multi-objective Decision-Making

The so-called seeking for fuzzy multi-objective decision-making refers to making decisions in case of multiple objectives that may conflict with each other and are incommensurable with each other. However, in reality, most of the cases are a kind of the multi-objective decision-making problem, for instance, the optimal design of weaponry, combat program evaluation and selection [3] etc. The model could be described as follows:

$$\max_{x \in X} \{f(x)\} \quad (1)$$

X is the decision space as well as the feasible region which is determined by characteristics and requirements of actual decision problems. X is the decision variable (while $x \in X$, we always call X the feasible solution).

$$f(x) = (f_1(x), f_2(x), \dots, f_m(x))^T \quad (2)$$

It represents the vector functions based on the amount of m , m is a positive integer. The vector functions could be benefit type or cost type. According to the actual situation, it just needs to be changed the positive or negative sign.

III. SAMPLE APPLICATION

A. Example and Process

The paper takes the operational effectiveness of military helicopters as the evaluate object and the configuration scheme which we need to confirm as the seeking for fuzzy multi-objective decision-making object. We take advantage of the world's typical top 20 gunships index as the knowledge base [4]. Partial information of the base is shown in Fig. 1.

C1	C3	C6	C7	C8	C9	C10	C11
COUNTRY	NAME	LONG	HEIGHT	DIAMETER	POWER	MAXSPEED	SPEED
US	RAH-66	14.48	3.39	11.9	3126	328	305
US	AH-64A	17.76	4.05	14.63	1940	365	265
US	AH-64D	17.7	4	14.6	1940	365	265
RU	K-50	15	4.9	14.5	4000	340	290
RU	K-52	15	4.6	14.5	4000	330	279
CN	WZ-10	14.15	3.85	13	3063	300	270
USA	AH-1	13.9	4.4	14.6	2082	313	278
ZA	CSH-2	18.7	5.1	15.6	3808	371	278
JP	AH-1J	13.9	4.4	14.6	1800	285	227
EU	SA341	11.9	3.1	10.5	600	310	270
EU	FAH-2	14	4.3	13	2570	320	230
RU	M-24	18.5	6.5	17.6	4450	335	295
IL	TZEFA	13.8	4.4	14.6	1800	290	227
EU	FAH-1	8.8	3	9.8	3300	242	205
EU	A-129	14.3	3.4	11.9	1500	313	240
BR	500MD	7.6	2.6	8	317	241	221
RU	M-17	18.4	5.7	21.3	4000	250	240
CN	WZ-9	13.7	4.5	11.9	1420	315	280
JP	CH-47J	15.5	5.7	2*18.3	4333	274	259
IL	UH-60A	19.5	4.8	16.5	3300	296	257
EU	SA330	15.5	4.92	15	3500	278	249
EU	SA321	23	6.7	18.9	3150	270	248
EU	SA319	12.8	3	11	700	220	190
MY	LYNX 300	13.5	3.7	12.8	1800	259	225

Fig. 1. Partial Information of knowledge base.

Based on rough set theory and the way of seeking for fuzzy multi-objective decision-making, the use of evaluating operational effectiveness will get the optimal configuration scheme. Assuming the requirement of more than 50% degree of completion, the process is shown in Fig. 2.

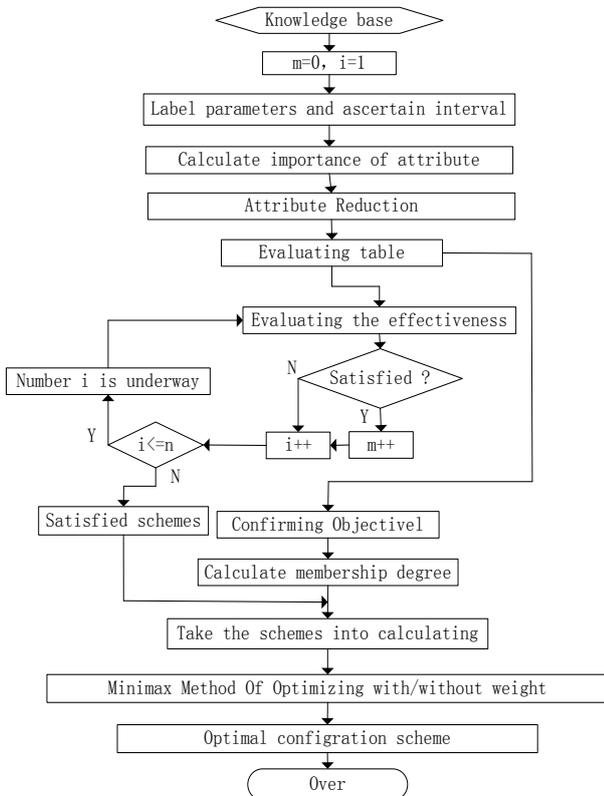


Fig. 2. The operational flowchart.

B. Evaluating Based on Fuzzy Set

1) Data preprocessing

Considering the fact that the preprocessing of attribute data [5] isn't the key part of the research, we wouldn't go into details. Through the analysis of rankings of the helicopters from the Internet and combining with the performance parameters above, we can get the comprehensive ranking of the military helicopters. According to the order of the ranking, we divide them into 4 groups with each one possessing 4 helicopters thus representing different effectiveness levels. The effectiveness levels correspond to main intervals of the degree of completion that are listed below:

$$a:(0.8,1],b:(0.8,1],c:(0.8,1],d:(0.8,1]$$

Selecting height of helicopters, power of engine, maximum rate of climbing and mounting system as objectives to be optimized, we label performance parameters as below:

$$[\text{High, Middle, Low}] = [a, b, c]$$

2) Attribute reduction

The following Table I is a decision table concerning the 20 gunships we mentioned.

$$U = \{X | X = x_i\} (i = 1, 2, \dots, 20)$$

The attribute set is $A = CUD$ and the condition attribute set is

$$C = \{\text{Height, Power, Climbing rate, System}\}.$$

The decision attribute set is

$$D = \{\text{Effectiveness Level}\}.$$

TABLE I: EFFECTIVENESS EVALUATING DATA TABLE

U	C ₇	C ₉	C ₁₂	C ₁₈	D
x1	b	b	b	a	a
x2	b	a	a	a	a
x3	b	a	a	a	a
x4	c	a	c	a	a
x5	b	a	a	a	a
x6	b	a	a	b	b
x7	b	b	a	b	b
x8	c	b	a	b	b
x9	b	b	b	b	b
x10	a	a	a	b	b
x11	a	a	a	a	c
x12	c	c	a	b	c
x13	b	b	b	b	c
x14	c	a	c	b	c
x15	c	c	b	b	c
x16	a	a	b	b	d
x17	b	a	b	c	d
x18	b	a	c	c	d
x19	a	a	c	c	d

In Table I C₇ = Height, C₉ = Power, C₁₂ = Climbing rate, C₁₈ = System, D = Effectiveness Level.

Divide U as required, and then we could get each positive region of D [6].

$$\begin{aligned} &POS_C(D); \\ &POS_{(C-\{c_7\})}(D); \\ &POS_{(C-\{c_9\})}(D); \\ &POS_{(C-\{c_{12}\})}(D); \\ &POS_{(C-\{c_{18}\})}(D). \end{aligned} \tag{3}$$

Calculate the degree of the importance of parameters C to the effectiveness level D.

$$\gamma_C(D) = \frac{|POS_C(D)|}{|U|} = 0.9 \tag{4}$$

Calculate the importance of effectiveness level of each condition attribute C_i.

$$\begin{aligned} \sigma_{c_7D}(C_7) &= \gamma_C(D) - \gamma_{C-\{c_7\}}(D) = 0 \\ \sigma_{c_9D}(C_9) &= \gamma_C(D) - \gamma_{C-\{c_9\}}(D) = 0.2 \\ \sigma_{c_{12}D}(C_{12}) &= \gamma_C(D) - \gamma_{C-\{c_{12}\}}(D) = 0.2 \\ \sigma_{c_{18}D}(C_{18}) &= \gamma_C(D) - \gamma_{C-\{c_{18}\}}(D) = 0.35 \end{aligned} \tag{5}$$

Normalize the degree of importance σ , and then we could get:

$$\begin{aligned} \lambda_7 &= 0 \\ \lambda_9 &= 0.267 \\ \lambda_{12} &= 0.267 \\ \lambda_{18} &= 0.467 \end{aligned} \tag{6}$$

Obviously, C₇ which represents the height of helicopters should be removed. That is, Power, Maximum rate of climbing and system have more contribution to the effectiveness level.

3) Rule reasoning and initial screening

Reasoning the information that has been processed, then

we can get:

$$\begin{aligned}
 F_1 &= (\{a(0.6, b(0.4)); \\
 &\quad \{a(0.6), b(0.2), c(0.2)); \\
 &\quad \{a(1)\}); \\
 F_2 &= (\{a(0.2), b(0.6), c(0.2)); \\
 &\quad \{a(0.8), b(0.2)); \\
 &\quad \{b(1)\}); \\
 F_3 &= (\{a(0.2), b(0.2), c(0.6)); \\
 &\quad \{a(0.4), b(0.4), c(0.2)); \\
 &\quad \{b(1)\}); \\
 F_4 &= (\{a(0.2), b(0.6), c(0.2)); \\
 &\quad \{b(0.4), c(0.6)); \\
 &\quad \{b(0.2), c(0.8)\});
 \end{aligned} \tag{7}$$

According to the 20 groups of scheme, which have already appeared the attribute sets, we begin initial screening by using the remaining 14 groups of schemes do not appear in the sets. Thus it is obvious of the effectiveness degree of group (b, b, b) as c ; the effectiveness degree of group (c, b, c) and (c, c, b) as c or less than c . Hence, the result of initial filtering is shown in Table II.

TABLE II: INITIAL SCREENING TABLE

No.	Group	Remove?	Reason
14	(a, b, a)	×	Can't Judge
15	(b, a, a)	×	Can't Judge
16	(a, a, c)	×	Can't Judge
17	(a, c, a)	×	Can't Judge
18	(c, a, a)	×	Can't Judge
19	(a, b, c)	√	By Group 3
20	(a, c, b)	√	By Group 3
21	(b, a, c)	×	Can't Judge
22	(c, b, a)	×	Can't Judge
23	(a, c, c)	√	By Group 3
24	(c, a, c)	×	Can't Judge
25	(c, c, a)	×	Can't Judge
26	(c, b, c)	√	By Group 12
27	(c, c, b)	√	By Group 12

After initial filtering, the remained groups, with the number of 14-18, 21-22 and 24-25, would be disposed by the rule reasoning. Calculate $D(F_j | X_j)$. For example, the result of the calculation is as follows when j is 14:

$$\begin{aligned}
 D(F_1 | X_{14}) &= 0.600; \\
 D(F_2 | X_{14}) &= 0.133; \\
 D(F_3 | X_{14}) &= 0.200; \\
 D(F_4 | X_{14}) &= 0.200.
 \end{aligned} \tag{8}$$

Therefore, we can get the result of the effectiveness evaluation [7]. That is, when the power of the engine is more than 3000 Horsepower, the maximum rate of climbing is between 450 and 590 meter per minute and the capability of the system is strong, the effectiveness degree of fighting of the military helicopter will achieve "a" at 60% confidence level, the degree will achieve "b" at 13.3% confidence level, the degree will achieve "c" at 20% confidence level and the degree will achieve "d" at 20% confidence level.

4) Improvement and secondary screening

The above effectiveness degree and the confidence level can only give the range of evaluating result which shows the

weakness, less intuitive and imprecise. In this paper, we improve a little bit in this aspect. Use the effectiveness degree and confidence level to acquire more accurate and intuitive specific values. In fact, select the information from the group with factor j as 1, 2, 3, 6, 7, and 8 as the partial knowledge case for correcting.

$$\begin{aligned}
 j = 1: \quad \delta_1 &= 1.9868 \\
 j = 2: \quad \delta_2 &= 1.7206 \\
 j = 3: \quad \delta_3 &= 1.2255 \\
 j = 6: \quad \delta_6 &= 1.3043 \\
 j = 7: \quad \delta_7 &= 1.4845 \\
 j = 8: \quad \delta_8 &= 0.9445
 \end{aligned} \tag{9}$$

The final completion and the average corrected coefficient are:

$$\text{Final completion} = \delta / \bar{\delta} \tag{10}$$

$$\bar{\delta} = \frac{1}{6} \sum_{i=1}^6 \delta_i = 1.39451 \tag{11}$$

TABLE III: EFFECTIVENESS EVALUATING TABLE

No.	Completion	Satisfied
14	0.514	√
15	0.713	√
16	0.487	×
17	0.471	×
18	0.588	√
21	0.516	√
22	0.418	×
24	0.392	×
25	0.376	×

Therefore when $j=14$, the completion is 0.541, which has already been corrected instead of the original 0.71645. And the result should satisfy the requirement with no less than 50% completion. For other 8 groups, repeat the steps above. Results are shown in Table III.

Through secondary filtering, the configuration scheme groups satisfying the above requirement are listed in Table IV.

TABLE IV: SECONDARY SCREENING RESULT

No.	Scheme Group
14	(a, b, a)
15	(b, a, a)
18	(c, a, a)
21	(b, a, c)

C. Multi-objective Decision-Making

1) Confirming the decision objective

In reality, most of cases belong to the multi-objective decision-making problem [8]. When researching the development plan of the military helicopters, the researchers are customarily concerned with various factors like motor performance, capability of the weapons, viability, research costs and time etc. In this paper, besides factors that have been mentioned above, we take cost and time of the research, that are cost type objectives, into consideration. According to the actual situation, we take the power of the engine (f_1), the maximum rate of climbing (f_2), the capability of the system (f_3), research costs (f_4) and time (f_5) as the factors that are closely related to the effectiveness value.

2) *Minimax method of optimizing*

The minimax method of optimizing, with an advantage of good usability, lays its stress on being conservative. The principle is that if $x_{j^*} \in X$ satisfies

$$\mu_{i j^*} = \max_{1 \leq j \leq n} \min_{1 \leq i \leq m} \{\mu_{ij}\} \quad (12)$$

x_{j^*} is the most suitable scheme. When there are more than two schemes that are satisfied with the formula, all these schemes are tied for the first in suitable. Remove x_{j^*} from X . Repeat the process so that we can get the ranking of the schemes.

The most suitable scheme which is sought by minimax method of optimizing can ensure the worst objective function to value of the largest. Obviously, it is a conservative method that can avoid making mistakes in game.

	x_1	x_2	x_3	x_4
f_1	3126	2082	760	1700
f_2	480	750	600	670
f_3	Strong	Strong	Strong	Weak
f_4	High	Middle	Middle	Low
f_5	Longer	Long	Middle	Short

Take caution that the evaluations of factors f_3, f_4, f_5 are fuzzy. That is, we use natural language [9] to describe the values of schemes, for examples:

(“Strong”, “Middle”, “Weak”)-(1,0.7,0.3), (“High”, “Middle”, “Low”)-(1,0.7,0.3) and (“Longer”, “Long”, “Middle”, “Short”)-(1,0.75,0.5,0.25). So that F could be translated into the relative membership degree matrix of the objectives. We know that f_1, f_2 and f_3 are benefit type objectives, while f_4 and f_5 are cost type objectives, the relative membership functions are:

$$\mu_{ij} = (f_{ij} / f_{i\max})^{p_i} \quad (i \in 0^{\text{Benefit type}}) \quad (13)$$

$$\mu_{ij} = \begin{cases} 1 - (f_{ij} / f_{i\max})^{p_i} & (f_{i\min} = 0) \\ (f_{i\min} / f_{ij})^{p_i} & (f_{i\min} \neq 0) \end{cases} \quad (i \in 0^{\text{Cost type}}) \quad (14)$$

Thus, we can get the updated matrix F' and μ .

	x_1	x_2	x_3	x_4
f_1	3126	2082	760	1700
f_2	480	750	600	670
f_3	1	1	1	0.3
f_4	1	0.7	0.7	0.3
f_5	1	0.75	0.5	0.25

	x_1	x_2	x_3	x_4
f_1	1	0.67	0.24	0.54
f_2	0.64	1	0.80	0.89
f_3	1	1	1	0.30
f_4	0.30	0.43	0.43	1
f_5	0.25	0.33	0.50	1

Obviously,

$$\max_{1 \leq j \leq 4} \min_{1 \leq i \leq 5} \{\mu_{ij}\} = \max \{0.25, 0.33, 0.24, 0.30\} = 0.33 = \mu_{52}$$

Therefore, the scheme 2, in which the power of the engine is “2082 Horsepower”, the maximum rate of climbing is “750 meter per minute” and the capability of the system is “Strong”, the research cost is “Middle” and the research time is “Long”, is judged to be the optimal scheme. Similarly, the ranking of the schemes is “Scheme 2”, “Scheme 4”, “Scheme 1” and “Scheme 3”.

3) *Minimax method of optimizing with different weights*

The method of optimizing above doesn't involve weights. However, most of problems are related to several factors [10]. For this reason, we introduce different weights to reflect the different emphasizes. The membership function could be defined as:

$$\mu_{i j^*} = \max_{1 \leq j \leq n} \min_{1 \leq i \leq m} \{\mu_{ij}^{\omega_i}\} \quad (17)$$

Therefore, the method, with the preference of each objective could be described, is called minimax method of optimizing with different weights [11]. Assume that the weight vector is

$$\omega = (0.1, 0.2, 0.4, 0.1, 0.2)^T.$$

Then, we can get:

$$\max_{1 \leq j \leq 4} \min_{1 \leq i \leq 5} \{\bar{\mu}_{ij}\} = \max \{0.76, 0.80, 0.87, 0.62\} = \bar{\mu}_{53}$$

Therefore, the scheme 3, in which the power of the engine is “760 Horsepower”, the maximum rate of climbing is “600 meter per minute” and the capability of the system is “Strong”, the research cost is “Middle” and the research time is “Middle”, is judged to be the optimal scheme. Similarly, the ranking of the schemes is “Scheme 3”, “Scheme 2”, “Scheme 1”, and “Scheme 4”.

Obviously, the final result given by “Minimax Method of Optimizing with Different Weights” is different from those given by the method without weights.

IV. ANALYSIS OF RESULTS

According to the operational effectiveness evaluation results, an optimal solution is got by analyzing multiple decision-making combinations which meet the effectiveness performance grades requirements with rough set theory. First, this paper adopted the minimax method and found that the second decision-making scheme combination was the optimal solution. Finds the optimal solution is the second decision-making scheme combination. After that the decision maker who combined their own preference and weights, used the minimax weight method again, getting the third decision scheme combination as the optimal solution. To further analyze different optimization results, the comparison result of scheme 2 and scheme 3 is shown in Table V.

According to Table V, scheme 2 and scheme 3 differ slightly in “the maximum rate of climb”, “the platform system” and “the development costs”. However, there are significant differences between “power” and “the development time”. Scheme 2 is lower than scheme 3 in “power” value and scheme 2 has longer “development time”. Thus, if optimization weight is not introduced, in the

“power”, scheme 2 has obvious advantages in the numerical solution over scheme 3. The final optimization result shows that scheme 2 is the optimal solution. When optimization weight is introduced, as decision makers would pay more attention to “the development time” in which scheme 3 is one level shorter than scheme 2 in length, the optimization result after weighted indicates that the scheme 3 is the optimal solution.

TABLE V: DIFFERENT SCHEMES

schemes attributes	Scheme 2	Scheme 3	coefficient
Power	2082	760	0.1
Rate of climbing	750	600	0.2
System	Strong	Strong	0.4
Cost	Middle	Middle	0.1
Time	Long	Middle	0.2

V. CONCLUSION

This paper proposes an effectiveness evaluation method based on rough set theory and takes the performance index set of top 20 gunships around the world nowadays as the knowledge base. First, this paper evaluates the operational effectiveness of decision-making based on various performance indicators combination and gets decision schemes satisfying the completion degree requirements after filtering. The decision schemes filtered are then solved according to the multi-objective decision optimization method. Experiments are done both before and after introducing the optimization weight to get different optimal decisions in different conditions with different emphasis. The screening stage is added in the experiment process, to some extent, it reduced the complexity of the follow-up process. Correction parameters are also added in the effectiveness evaluation completion confirmation, which could be helpful to get the rough evaluation result. The paper generally realizes the combined application between the rough set theory and multi-objective decision-making optimization in the operational effectiveness evaluation.

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Zhenghua Cui was born in Jilin, China in 1990. He is with the National University of Defense Technology, Changsha, Hunan, China. He has got the academic master degree of engineering. His research area is task planning technology.



Jian Huang was born in Zhejiang, China in 1973. She is with the National University of Defense Technology, Changsha, Hunan, China. She has got the academic doctor degree of engineering. She is a D.E. Ph.D. supervisor. Her research area is task planning technology.