

Optimal Decision Making Method for Multi Criteria Problems

Roghayeh Zamani and Pirayeh Yousefi

Abstract—Decision making is one of the essential and important fields nowadays, by integrating decision making on multi criteria and fuzzy logic, there will be a satisfactory theoretical framework for system evaluation. There exist several methods for multi parametric decision making. In this paper we will introduce a new way and optimal method for multi parametric problems by taking Fuzzy AHP method into consideration. In this method, it is planned to calculate the weight of criteria by the help of AHP analytical steps. Then we will identify the best alternative by the help of the proposed method.

Index Terms—Multi criteria decision making (MCDM), AHP, fuzzy AHP.

I. INTRODUCTION

Optimal models are of great importance in decision making and there are employed in significant industrial issues. Multi criteria decision making has been introduced as one of the best methods in 1970. In recent years researchers have become interested in multi criteria decision making models. In this kind of decision making, instead of using a single method for evaluating Optimacy, they employ multiple criteria. Fuzzy set theory introduces an optimal method for decision making in MCDM issues to obviate complexity in human manner of think.

There are several ways for MCDM that one of which is AHP. This method is introduced by Saaty in 1980 for the first time. AHP is a method for ranking the alternatives in order to select the best one in case of having multiple criteria. In AHP, selecting between alternatives is carried out by the help of intelligent comparison. In a conscious comparison, the decision maker compares both of the solutions by taking one criteria into consideration. AHP is a method that is suitable for complicated systems and selects one option among all. AHP is established on considering a problem based on hierarchical order and lets analysts to consider different dimensions of the problem in a hierarchical structure like a tree, so it can convert complicated decision making to simple comparison and ranking to reach logical results [1], [2].

In AHP we should consider the qualitative and quantitative aspects of the issue.

In this method, the problem is considered in a hierarchical order and in this classification, the aim is at the top; the criteria are in middle and the alternatives are at the end of the

hierarchy [3]-[5].

Here all of the components are compared with their counterpart criterias. The analytic method tries to calculate index vectors until the final common coefficient vector is determined for all of the alternatives. The inputs of the final weight coefficient vectors, is the value of every alternative for every goal which is placed at the top of the hierarchy [2], [5].

In sum, the steps employed in AHP, are as followings:

- 1) Specifying the decision making criterion like a goal hierarchy. This hierarchy has different levels in which, the goal is on the top, criteria and subcriteria are in middle and the alternatives are in the down.
- 2) It evaluate the weight of the criteria and subcriteria and alternatives, as a function of their importance, which shows their importance for the upper element. For achieve this goal, AHP does simple peer to peer comparisons, to specify the weights and the analyst can focus on two factor at the same time.
- 3) After expanding the comparison matrix, a priority vector for getting the weight of matrix elements can be calculated. This index vector is the normalized version of the matrix.

We use AHP instead of multi criteria techniques because:

- 1) The quality and quantity criteria are considered in decision making.
- 2) Many of the criterias are taking into consideration.
- 3) A flexible hierarchy which is accommodate to the problem is built.

In this paper, Section II refers to review of fuzzy AHP, Section III explains the proposed method, Section IV represents numerical example and the last section represents the conclusion.

II. FUZZY AHP

In order to cooperating of ambiguity of human reflection, Zadeh represented the theory of fuzzy set that tended to rationalism, incertitude because of ambiguity and inaccuracy. One of current problem in fuzzy set theory is its potency in representing of ambiguous data. This theory allows mathematical users to plan for using of fuzzy domain. The fuzzy set is a group of variants that have membership continuum. This set is determined by the help of the parametric membership functionality, which it assigns a degree of membership between 0, 1 to each of them. The symbol " \sim " on top of the function, represents the fuzzy set. Therefore, \tilde{a} , \tilde{p} and \tilde{m} are all of fuzzy group.

The membership performance of these groups designed by $\mu(x|\tilde{p})$. Fig. 1 represents a triangular figure (TFN).

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The TFN determined as $(\frac{m_1}{m_2}, \frac{m_2}{m_3})$ or (m_1, m_2, m_3) . The parameters of m_3, m_2, m_1 is smallest, middle and largest possible value respectively. The possible largest value is determined of a fuzzy event.

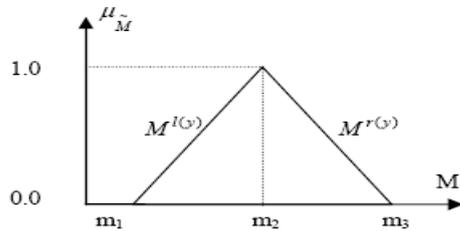


Fig. 1. TFN numbers (triangular fuzzy numbers).

The fuzzy AHP is an extended version of AHP that makes the data fuzziness possible. Understanding of the fuzzy AHP is easy and management of the quality and quantity of data in multi data decision making is easy [3], [5].

In AHP steps the result of peer to peer comparisons in final matrix are fuzzy numbers that are explained by the designer.

III. THE NEW METHOD OF FUZZY MULTI CRITERIA DECISION MAKING

In this method we use triangular fuzzy number. To calculate the weights of triangular numbers, we used buckly geometric average for quadruple numbers as well which is modified for fuzzy triangular numbers. At first, we organize the problems as hierarchical structure and after determining the given levels, criteria and alternatives, obtain comparative analogy for any level with regard to top level goal. After calculation of matrix, the geometric mean any row of matrix was obtained and calculated based on them, the values of w_i, wn_i for any matrix. Then we divide the sum of the achieved weights and criteria by the sum of the sum of the weight achieved for criteria. We determine the least value as the fuzzy number for that index. Then by using of (4), (5) we calculate $\mu(u_i)$ for any of numbers and select the optimal one which has extreme value among indexes, as the best alternative. The given algorithm phases includes:

- 1) We obtain the comparative analogy matrix A from DM as $\tilde{x} = (a_{ij}, b_{ij}, c_{ij})$.
- 2) We calculate geometric mean of any row:

$$h_i = \left\{ \tilde{x}_{i1}(\cdot)\tilde{x}_{i2}(\cdot)\dots(\cdot)\tilde{x}_{in} \right\}^{\frac{1}{n}} \quad (1)$$

where \tilde{x}_{ij} is the index I from fuzzy number for j th criteria, h_i is i th index geometric mean.

- 3) We calculate the weights w_i for any row of comparative matrix as follow :

$$w_i = \frac{h_i}{h_1(+)\dots(+)\dots(+)\dots(+)\dots(+)\dots(+)\dots(+)\dots(+)} \quad (2)$$

where w_i is i th index weight for criteria matrix.

- 4) After calculating the weights for criteria matrix, we calculate wn_{ij} weights by (1) and (2). wn_{ij} is the weight of i th index for j th criteria for j th criteria.
- 5) We calculate the u_i values by using (3) after measuring of wn_{ij}, w_i weights.

$$\forall i \in m \quad u_i = \min_{j=1}^n ((wn_{ij} + w_j) / \sum_{j=1}^n w_j) \quad (3)$$

where u_i is obtained fuzzy number for any alternative.

- 6) We calculate the $\mu(u_i)$ value by (4) for alternatives after obtaining u_i and select the best alternative from obtained value:

$$\mu(u_i) = \min(s_{i,j}) \quad (4)$$

$$S_{i,j} = \frac{(2 * u_i * u_j)}{(u_i + u_j)^2 - (u_i * u_j)} \quad (5)$$

where $S_{i,j}$ is the i 'th fuzzy number to j fuzzy number and $\mu(u_i)$ is obtained mathematical value for any alternative.

We select the extreme value as the most proper alternative after calculation of given value. We rate the alternatives by $\mu(u_i)$ values. In the following, the mathematic example is given and the result of this method contrasted with two other method i'e fuzzy MCDM and topsis.

IV. EXAMPLE

Use either A director is going to select most proper project for conducting with regard to given criteria. The criteria include: Net Present Value (NPV), Capital Return Rate (ROR), Benfit Charge (BC), PayBack date (PB). Now, we are going to select the proper one of six available projects for conducting with regard to his criteria [4].

By using of given method, the first step is forming rated structure and making decision matrix for criteria and alternatives. The Tables I and II represent the criteria matrix for criteria and indexes matrix for one of criteria (ROR). We calculate the weights for any of them after forming matrix by (1).

$$\begin{aligned} w_1 &= (.1525, .2856, .4826) \\ w_2 &= (.0794, .1580, .3052) \\ w_3 &= (.1789, .3396, .6555) \\ w_4 &= (.1662, .2168, .4188) \end{aligned}$$

$$\begin{aligned} wn_{1,1} &= (.0572, .2853, .3.0207) \\ wn_{1,2} &= (.0485, .08000, .9010) \\ wn_{1,3} &= (0, .05990, .4213) \\ wn_{1,4} &= (0, .1763, .17.9248) \end{aligned}$$

The given calculations are mentioned for two matrixes. For

other matrixes, we calculate $wn_{i,j}$ values as same way. Now, we calculate the u_i values for each alternative by 3 and $\mu(u_i)$ values for all of alternatives that, has been recorded on Table III.

As has been seen in given table, the project6 has extreme value and is the best select for him. The prioritizing of the alternatives as follows:

$$P_6 > P_3 > P_2 > P_1 > P_4 > P_5$$

The given example was calculated by fuzzy MCDM and TOPSIS that is one of the mathematical methods in multiple decision making metho. Table IV shows the results of our experiments.

Fig. 2 represents the normalized decision numbers for various alternatives by giving three methods. As seen in Fig. 2 at any of these methods, P6 has been selected as the best alternative and the priority of alternatives is as follows:

The new method: $P_6 > P_2 > P_1 > P_3 > P_4 > P_5$

TOPSIS: $P_6 > P_5 > P_3 > P_2 > P_4 > P_1$

MCDMFuzzy: $P_6 > P_2 > P_3 > P_1 > P_4 > P_5$

If we compare the proposed method with MCDM, we see that they have more accomodation about giving the priority to alternatives. So according to AHP's advantages and its efficiency in the problems, our proposed method is appropriate and simple.

TABLE I: CRITERIA COMPARE MATRIX

	ROR	PB	NPV	CB
ROR	(1,1,1)	(1,2,3)	(.75,1,1.25)	(.75,1,1.25)
PB	(.33,.5,1)	(1,1,1)	(.25,.5,.75)	(.5,.75,1)
NPV	(.8,1,1.33)	(1.33,2,4)	(1,1,1)	(1,2,3)
BC	(.8,1,1.33)	(1,1.33,2)	(.33,.5,1)	(1,1,1)

TABLE II: COMPARE MATRIX FOR ROR

ROR	P1	P2	P3	P4	P5	P6
P1	(1,1,1)	(.0285, .2785, .5285)	(.7072, .9572, 1.2072)	(.5422, .7922, 1.0422)	(.2587, .6787, .9287)	(.456, .7060, .956)
P2	(.6585, .9085, 1.1585)	(1,1,1)	(.2354, .4854, .7354)	(.7095, .09595, 1.2095)	(.5077, .7577, 1.0077)	(0, .0318, .0636)
P3	(.027, .1270, .227)	(.7075, .9575, 1.2075)	(1,1,1)	(.4057, .6557, .9057)	(.4931, .7431, .9931)	(.0269, .2769, .5269)
P4	(.6634, .9134, 1.1634)	(.7149, .9649, 1.1249)	(.0419, .1419, .2419)	(1,1,1)	(.1422, .3922, .6422)	(0, .0462, .0924)
P5	(.3824, .6324, .8824)	(.0576, .1576, .2576)	(.1718, .4218, .6718)	(.5991, .8491, 1.0991)	(1,1,1)	(0, .0971, .1942)
P6	(0, .0975, .195)	(.7206, .9706, 1.2206)	(.6657, .9157, 1.1657)	(.144, .3940, .644)	(.0712, .1712, .2712)	(1,1,1)

TABLE III: THE NORMALIZED VALUE OF FINAL SCORE OF PRESENTED METHOD

	New method	fuzzy MCDM	TOPSIS
P1	0.68444	.5444	.378043
P2	0.75605	.5901	.496176
P3	0.7857	.5773	.538381
P4	0.6450	.4780	.438027
P5	0.63957	.4634	.54468
P6	0.85501	.6029	.604535

TABLE IV: THE NORMALIZED VALUE OF FINAL SCORES OF METHODS

alternative	score
Project1	0.68444
Project2	0.75605
Project3	0.7857
Project4	0.6450
Project5	0.63957
Project6	0.85501

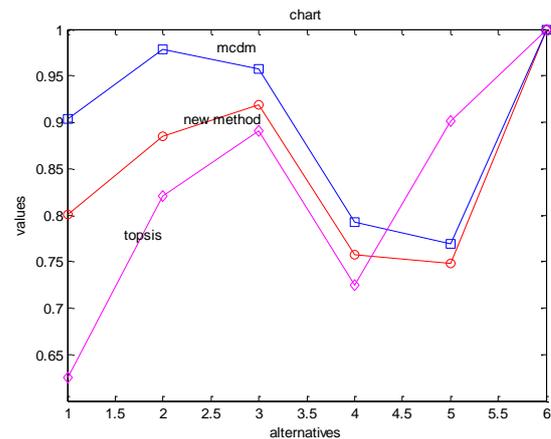


Fig. 2. Normalized value of three methods.

V. CONCLUSION

In this paper, a new method is proposed based on fuzzy AHP Method in which, the numbers and data is as triangular fuzzy numbers. The advantage of this algorithm is that, it is applicable for any types of MCDM problems. This method is reasonable for MODM problems that in them, there isn't available accurate decision matrix, and has agreeable application for problems that, criteria is multi layer or is used of several DM for decision making. The other advantage of it, related to simplicity of calculation and low action time that could be has reasonable application in real time systems. The obtained results of given method was contrasted with MCDM and Topsis. The results indicate that, it could be applied in fast calculations in real time problems.

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