EOTs Based Fixed Order Controller Design and Performance Analysis

Faizullah Mahar, Saad Azhar Ali, Ayaz Hussain, and Zohaibdin Bhutto

Abstract—Evolutionary optimizations techniques (EOTs) are stochastic search techniques that direct a population of solutions towards best possible results by using the natural principles. In recent years, these algorithms have grown to be an accepted optimization tool for many areas of scientific and engineering research, together with control system engineering design. Significant research exists concerning evolutionary algorithms to control system design and robustness performance analysis of controller. But, little work has been done with evolutionary optimization algorithms because of the problems related with robustness performance in early period of the evolution of controllers. Moreover, until recently the robustness performance of controllers based on evolutionary algorithms has not been researched in stipulate. The scope of this research covers generalized robustness performance condition and detailed analysis of the resultant control system.

Index Terms—Evolutionary algorithm, optimization, robustness, performance and controller.

I. INTRODUCTION

A successfully designed control system should be always able to maintain stability and performance level in spite of uncertainties in the system. Due to its importance, however, the research on robust design has been going on all the time. A breakthrough came with the pioneering work by [1] and [2] on the theory, now known as the $H_{\infty}$ control theory. If a control system is working under uncertainty, the control system is robust. A controller that performs suitably in the presence of plant parameter variation is said to be a robust [3].

The aim in control system design is to design a controller, which connected to a system, provides a desired behavior of the control system [4]. Fixed order robust controllers have become interesting area of researchers because of simplicity and acceptable controller order [5].

Regardless of the existing elegant methods of optimal and robust control engineers complain about the gap between theories and practice in control systems many of the designed techniques cannot incorporate realistic constraints such as fixed structure [6]. Interesting ways solving the problems are heuristic algorithms. PID controller is one of the most primitive developed control tactics [7].

Mostly, the controllers used in industrial process are PI or PID controllers.

Unfortunately, tuning of control parameters of these controllers for achieving both robustness and performance specifications is difficult. To overcome this problem, the approaches to design a robust control for structure specified controller were proposed in [8].

In this research, the performance and robust stability conditions of the designed system satisfying the $H_{\infty}$ loop shaping are formulated as the cost function in the optimization problem. EOTs (GA, PSO and IA) are adopted to solve this problem and to achieve the control parameters of the proposed controller.

For this specific control system, LSDP is combined with a PI control structure, it can be guaranteed that the stability and performance robustness are satisfied and the control structure is as simple as a PI controller. Simulation results validates that the robustness and performance of the proposed robust controller design approach.

The primary motivation of this work is to present a complete treatment on the design and application of EOTs for CF minimization in the presence of uncertainties. Also treats the robustness performance analysis.

The remainder of this paper is organized as follows. Overview of EOTs is presented in section II. Loop shaping design procedure is given in section III. Detailed design procedure of the proposed schemes is discussed in section IV. Section V presents simulation results, robustness performance analysis is presented in section VI and conclusion is placed in section VII.

II. EVOLUTIONARY OPTIMIZATION TECHNIQUES

Optimization has been essential component of many engineering technology fields, a number of approaches exist to get an optimal behavior in a process of plant. EOTs have emerged as a contestant due to its flexibility effectiveness in variety of optimization applications [9].

GA is a search method, starts the process with randomly initialization of population of individuals. Then the fitness of each individual is calculated [10]. The transmission of one population to next takes place by means of the genetic operators such as selection, crossover and mutation. This selection process chooses the fittest individual from the population to continue in the next generation. Cross over randomly chooses a locus and exchanges the subsequences before and after that locus between two chromosomes to create two offspring. Mutation operator randomly flips some of bits in the chromosome [11].

Mostly, researchers use GA as a search technique in combination with $H_{\infty}$ loop shaping design procedure (LSDP)
[12]. In $H_{\infty}$ LSDP pre-compensator and post-compensators are required [13]. GA is used to optimize the cost function and controller parameters that define the structure [14].

In recent years, particle swarm optimization (PSO) procedure appeared as promising technique for managing the optimization problems. Recently PSO is used to design robust controllers for dynamics of multi-machine system.

PSO is population based evolutionary technique that has many key advantages over other optimization techniques and it is general purpose optimizer that solves the wide range of optimization problems thus the PSO can be adapted to various categories of optimization [15].

In IA, antigen represents the problem to be solved and an antibody set is generated where each number represents a candidate solution [16]. Also an affinity is the fit of an antibody to the antigen. The role of antibody is to eliminate the antigen.

In IA n number of antibodies generated randomly [17]. While affinity of all antibodies is known new population is generated through three steps: replacement, cloning and hyper-mutation. In replacement step low antibodies are replaced those with highest affinity are selected by cloning and hyper-mutation is applied where the mutation rate is inversely proportional to its affinity [18].

III. LOOP SHAPING DESIGN PROCEDURE

This approach requires only a desired open loop shape in gain as a rule at low frequency range is necessary. There are two steps are in this procedure. In first step two weighting functions $W_1$ and $W_2$ are specified to shape the original plant. The singular values of the shaped plant satisfy the closed-loop performance requirements [19].

$$G_s = W_1G_0W_2$$

The weighting functions are chosen as

$$W_1 = \frac{0.8s + 4}{s + 0.001}, \quad W_2 = 1$$

where I is the identity matrix, with these weighting functions band width of the desired control system is increased from 3.4 rad/sec to 4.2 rad/sec, drastically improves the performance and robustness.

In the second step, the controller $K_\infty$ is synthesized and final controller $K(s)$ is constructed by multiplying $K_\infty$ with weighting functions as shown in Eq.(1).

$$K(s)_{\text{final}} = W_1K_\infty W_2$$  \hspace{1cm} (1)

IV. FIXED ORDER ROBUST CONTROLLER DESIGN

In this proposed scheme, PSO, GA and IA are used to minimize the cost function. From the procedure discussed previously the transfer function of the identified plant model is shown in Eq.(2).

$$G(s) = \frac{551.3e^{(-0.12s)}}{(s^2 + 43.26s + 536.9)}$$

The performance and robust stability conditions of the designed system satisfying $H_{\infty}$ loop shaping design procedure are formulated as cost function.

A. Controller Structure Selection

The structure $K(p)$ of controller is specified before starting the optimization process. A set of controller parameters $p$ is evaluated to minimize cost function (CF).

In this paper, EOTS (GA, PSO and IA) are adopted to find the optimal values of controller parameters $p^*$ in stabilizing controller $K(p)$ such that $\|T_{\infty}\|$ is minimized [19].

$$K(p) = W_1K_\infty W_2$$  \hspace{1cm} (3)

$$\|T_{\infty}\| = \left\| \frac{I}{W_1K(p)} \left[I + G_iW_{1}^{-1}K(p)^{-1}M_s^{-1} \right] \right\|_{\infty}$$  \hspace{1cm} (4)

B. Proposed Scheme by Using PSO

The steps of the proposed scheme are as follows:

Step-1, Initialize several sets of population parameters $p$ as population of particles, where $p$ is considered as a vector of controller parameters.

Step-2, Specify the controller structure evaluates CF of each particle.

Step-3, At each generation the velocity of each particle and position of the next is calculated.

Step-4, If current iteration is less than maximum iterations then stop, go to step 3.

Flow chart of the proposed scheme is shown in Fig. 1.

![Flow chart](image-url)
C. Proposed Scheme by Using IA

The steps for designing the fixed order robust controller using immune algorithm are:

Step-1, Generate initial sets of parameters \( p \) as population of antibodies

Step-2, Specify the controller structure \( K(p) \) where \( p \) is considered for each string of antibodies as a vector of controller parameters, evaluate CF of each antibody using eq. (4).

Step-3, Best antibody in the present problem is chosen as an antigen, which has minimum CF, affinity of each antibody can be calculated by using Eq. (5).

\[
Affinity = \frac{f(antigen)}{f(antibody)}
\]  

Flow chart of the proposed scheme by using IA is shown in Fig. 2.

D. Proposed Scheme by Using GA

The steps of the proposed scheme can be described as follows:

Step-1, Initialize several sets of parameters \( p \) as first generation, where \( p \) is considered as a vector of controller parameters.

Step-2, Specify the controller structure and evaluates the CF of each chromosome using Eq. (4).

Step-3, Select chromosomes with lowest CF as solution in present generation and apply GA operators.

Step-4, if current generation is less than the maximum generation, create a new population by using GA operators and go to step 3, if current generation is maximum generation then stop.

Finally, check the performance in both frequency and time domain. Flow chart of the proposed scheme by using GA is shown in Fig. 3.

V. SIMULATION RESULTS

The fixed order robust controller design by using EOTs has been simulated to predict performance of the proposed approach. By using the LSDP the final controller is obtained as:

\[
K(s) = \frac{310.1s^2 + 3308s + 8221}{s^4 + 43.26s^3 + 537s^2 + 1.074s + 0.05369}
\]  

The controller obtained by conventional techniques Eq.(6) is of 4\(^{th}\) order and its structure is complex and difficult to implement practically.

A. Investigation by Using PSO

The controller structure is expressed in Eq. (7)

\[
K(p) = K_p + \frac{K}{s}
\]  

The simulation was carried out using representation of particles. The size of initial population is 10 particles. Algorithm converged in 3\(^{rd}\) iterations, and gave optimal cost function of 1.4744. Fig.4 shows the plot of convergence of cost function versus iterations of PSO.
The optimal solution of controller parameters was obtained which has satisfied stability margin of 0.678, it shows that PSO can find optimal solution. The computed optimal values of controller parameters are shown in Eq. (8).

\[
K(p)^* = 0.400 + \frac{1.00}{s}
\]  

(8)

The step response of the control system with optimized controller parameters by using PSO is shown in Fig. 5; the step response presents rise time 1.25 sec., about 2% overshoot and the settling time is about 1.23 sec.

\[K(p)^* = 0.400 + \frac{1.00}{s}\]

B. Investigation by Running GA

The simulation was carried out by running GA. The size of initial population was 10; tournament selection and single bit wise mutation was used. GA converged on 4th generation and gave optimal CF of 1.3960. Fig. 6 shows the plot of convergence of CF versus generations of GA. The optimal solutions of specified controller parameters were obtained on 4th generation, which has satisfied stability margin of 0.716. It shows that GA can find optimal solution of fixed order controller parameters in several generations. Obtained optimal values of controller parameters are shown in Eq. (9).

\[
K(p)^* = 0.900 + \frac{0.05}{s}
\]  

(9)

The step response of the control system which was determined by optimized controller parameters by using GA is shown in Fig. 7, the step response present 0.66 seconds rise time, 14% overshoot and the settling time 2.73 seconds, the results obtained clearly shows the effectiveness of proposed scheme.

\[K(p)^* = 0.9991 + \frac{0.584}{s}\]

C. Investigation by Using IA

Afterward, investigation has been performed for PI controller as a fixed structure controller. \(k_p\) and \(k_i\) are the controller parameters that would be evaluated by using IA. The specific controller structure is expressed in Eq. (7). The simulation was carried out using IA with representation of antibodies. The size of initial population was 10 antibodies, colonial affinity was calculated and single bit mutation was used. IA converged on the 3rd iteration and gave the optimal CF of 1.3095. Fig. 8 shows the plot of convergence of cost function versus iterations of IA.

\[K(p)^* = 0.9991 + \frac{0.584}{s}\]

The optimal solutions of controller parameters were obtained on 3rd iterations, which has satisfied stability margin of 0.763. It shows that IA can find a global optimal solution of fixed order controller parameters in little iteration for this particular problem. Obtained optimal values of controller parameters are shown in Eq. (10).

\[K(p)^* = 0.9991 + \frac{0.584}{s}\]  

(10)

The closed loop step response of the control system with optimized controller parameters by using IA is shown in Fig. 9. The step response presents rise time 1.06 sec., 2% overshoot and the settling time is about 2 sec. the results obtained clearly shows the effectiveness of proposed scheme.
Controller parameters were obtained experimentally by using Z-N techniques based on the unit step response of the nominal plant. The controller parameters obtained using Z-N technique is shown in Eq. (11).

\[ K(p) = 4.495 \frac{12}{s} \quad (11) \]

The closed loop step response of the system is shown in Fig.10 present an over shoot of about 58 %, rise time 0.25 sec. and settling time 3.7 sec.

The closed loop step response of the system is shown in Fig.10 present oscillation and stability problem.

VI. PERFORMANCE ANALYSIS

In order to analyze the performance the designed optimal controller Eq. (8), Eq. (9), Eq. (10) and Eq. (11) obtained by running PSO, GA, IA and Z-N method respectively was implemented to control the perturbed plant Eq. (12).

\[ G_3(s) = \frac{551.1e^{-0.12s}}{(8s^2 + 43.26s + 536.9)} \quad (12) \]

The result shown in Fig.11, demonstrates that the designed controller from the proposed scheme using EOTs have reasonably good performance and robustness.

<table>
<thead>
<tr>
<th>TABLE I: COMPARISON BETWEEN OPTIMIZED PARAMETERS.</th>
</tr>
</thead>
<tbody>
<tr>
<td>para</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>( K_p )</td>
</tr>
<tr>
<td>( K_i )</td>
</tr>
<tr>
<td>CF</td>
</tr>
</tbody>
</table>

The CF value and the parameters of the controller optimized using GA and IA were compared to that obtained by using Z-N method. Results shown in Table-I, indicates that EOTs gave much better solutions than conventional \( H_\infty \) and Z-N. The PI gains obtained by using Z-N method are quite high values as compared to PSO, GA and IA optimization algorithms. Moreover, the best CF value is obtained from IA as compared to GA and PSO.

<table>
<thead>
<tr>
<th>TABLE II: COMPARISON BETWEEN PSO, GA, IA AND Z-N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>( S_t ) in sec.</td>
</tr>
<tr>
<td>( R_t ) in sec.</td>
</tr>
<tr>
<td>%Overshoot</td>
</tr>
</tbody>
</table>

From above comparisons shown in Table-II, Z-N, PSO and GA have higher settling time than IA. So tuning PI controller for plant using IA is more optimal than GA and PSO. The controller optimized with IA has provided much better response than controller optimized with GA and PSO. Moreover, the PI controller tuned by ZN has a maximum overshoot about 58 %.

VII. CONCLUSION

An appropriated performance weight \( W_1 \) that satisfying the time domain specifications and robustness is evaluated by, PSO, GA and IA. Based on the adequate weight selection, the responses from the designed controllers from proposed EOTs schemes are better than the responses from the PI tuned by ZN.

This proposed technique is an alternative method which directly considers the performance specifications and robustness in the design and in which the structure of controller is not restricted to PID. The controller \( K(p) \) can be replaced by any fixed-structure controller and the proposed algorithms can still be applied functionally.
REFERENCES


